

Elementary Analysis of Distortion in Diode Detection of Amplitude Modulation

Introduction

This analysis considers the distortion produced by diode detection of amplitude modulation. A typical detector consisting of series diode followed by a simple RC lowpass filter is analyzed.

Theoretical Analysis

The diode characteristic is given by the Shockley equation:

$$(1) \quad I_d = I_{rSS} [e^{(V_d/2V_T)}]$$

The exponential power series is given by:

$$(2) \quad e^x = 1 + x + x^2/2! + x^3/3! + x^4/4! + \dots + x^n/n!$$

where, in the case of a diode, $x = V_d/2V_T$ and $V_T \sim 26$ mV at 300 deg K for a silicon diode.

Examining the x^2 component of Eq. 2, for the diode detector:

Let w_c be the carrier frequency, w_m be the modulating frequency, w_1 be the upper sideband ($w_c + w_m$), and w_2 be the lower sideband ($w_c - w_m$).

The input to the diode is given by:

$$(3) \quad \cos w_c t + m/2(\cos w_1 t + \cos w_2 t), \text{ an AM signal where } m \text{ is the modulation index.}$$

The diode output is given by:

$$(4) \quad d(t) = [\phi(t)]^2 = [\cos w_c t + m/2(\cos w_1 t + \cos w_2 t)]^2$$

Expanding and simplifying:

$$(5) \quad d(t) = \cos^2 w_c t + m/2(\cos w_c t * \cos w_1 t + \cos w_c t * \cos w_2 t + \cos w_c t * \cos w_1 t + \cos w_c t * \cos w_2 t) + m^2/4(\cos^2 w_1 t + \cos^2 w_2 t + \cos w_1 t * \cos w_2 t + \cos w_1 t * \cos w_2 t)$$

$$(6) \quad d(t) = \cos^2 w_c t + m(\cos w_c t * \cos w_1 t + \cos w_c t * \cos w_2 t) + m^2/4(\cos^2 w_1 t + \cos^2 w_2 t) + m^2/2(\cos w_1 t * \cos w_2 t)$$

$$(7) \quad d(t) = \cos^2 w_c t + m/2[\cos(w_c + w_1)t + \cos(w_c - w_1)t + \cos(w_c + w_2)t + \cos(w_c - w_2)t] + m^2/8 (2 + \cos 2w_1 t + \cos 2w_2 t) + m^2/2[\cos(w_1 + w_2)t + \cos(w_1 - w_2)t]$$

$$(8) \quad d(t) = \cos^2 w_c t + m/2[\cos((w_c + (w_c + w_m))t + \cos((w_c - (w_c + w_m))t + \cos((w_c + (w_c - w_m))t + \cos((w_c - (w_c - w_m))t)] + m^2/8 [2 + \cos (2w_c + w_m)t + \cos (2w_c - w_m)t] + m^2/2[\cos ((w_c + w_m) + (w_c - w_m))t + \cos ((w_c + w_m) - (w_c - w_m))t]$$

$$(9) \quad d(t) = \cos^2 w_c t + m/2[\cos(2w_c + w_m)t + \cos w_m t + \cos(2w_c - w_m)t + \cos w_m t] + m^2/8 [2 + \cos (2w_c + w_m)t + \cos (2w_c - w_m)t] + m^2/2[\cos 2w_c t + \cos 2w_m t]$$

The typical diode detector is followed by a lowpass filter (typically an RC type, that passes only the audio frequencies). Thus, all w_c related terms in Eq. 9 are removed or filtered out giving:

$$(10) \quad m(t) = m^2/4 + m * \cos w_m t + m^2/8[\cos 2w_m t]$$

Here, the $m^2/4$ is the DC term, the $m * \cos w_m t$ term is the desired audio signal, and the $m^2/8$ term is the distortion term (second harmonic distortion to be exact). It is clear that the harmonic distortion will increase as the modulation level increases because the distortion is increasing as the square of the modulation index, and the fundamental (desired audio) is increasing linearly with the modulation index. Also, for simplicity's sake, the higher order terms (3rd, 4th, 5th, harmonic distortion components) were not considered. But in actuality, they would contribute to the overall total harmonic distortion.

Total Harmonic Distortion (THD) expressed as a percentage is given by:

$$(11) \quad \text{THD (\%)} = (P_h/P_f) * 100, \text{ where } P_h \text{ is the power in the harmonics, and } P_f \text{ is the power in the fundamental.}$$

Thus, the diode detector THD is given by:

$$(12) \quad \text{THD (\%)} = (m^2/8)^2/m^2 * 100, \text{ or } (m^2/64) * 100$$

Calculating the THD for various modulation indices gives:

Modulation Index (%)	THD (%)
10	0.015625
20	0.0625
30	0.140625
40	0.25
50	0.390625
60	0.5625
70	0.765625
80	1.0
90	1.265625
100	1.5625
150	3.515625
200	6.25

Table 1 - Theoretical Total Harmonic Distortion Versus Modulation Index

The typical diode detector consisting of series diode followed by a simple RC lowpass filter will produce a distorted output due to the inherent non-linearities of the diode. The distortion increases exponentially with the modulation index.

Empirical Analysis

To verify the theoretical analysis, the diode detector circuit shown in Figure 1 was constructed. An AM signal with various modulation indices was input and the THD on the output was measured. The test equipment configuration is shown in Figure 2. The results are shown in Table 2.

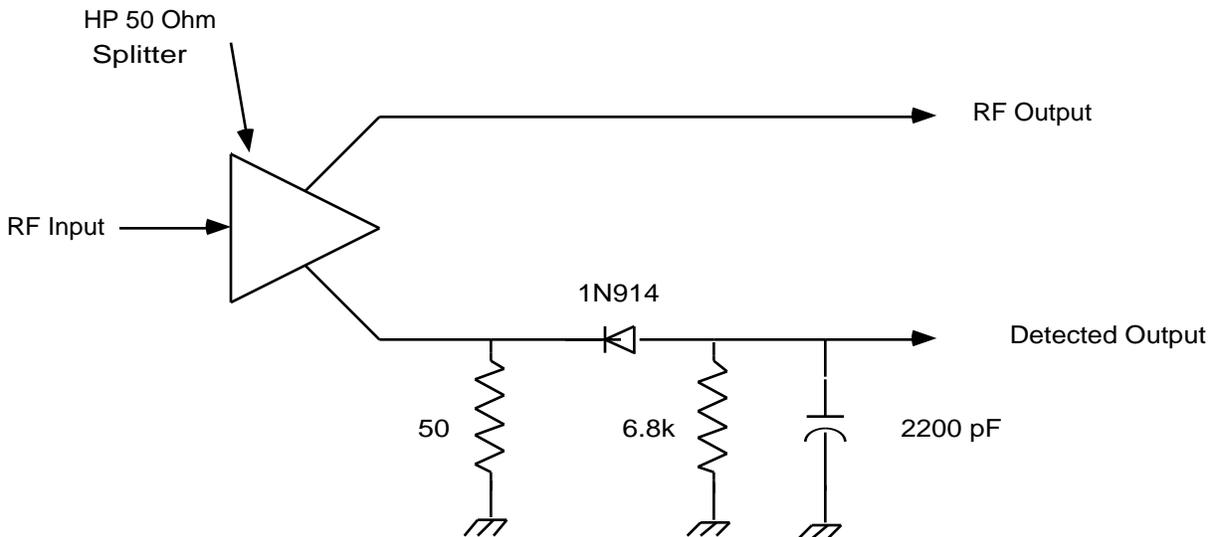


Figure 1 - Diode Detector Test Circuit

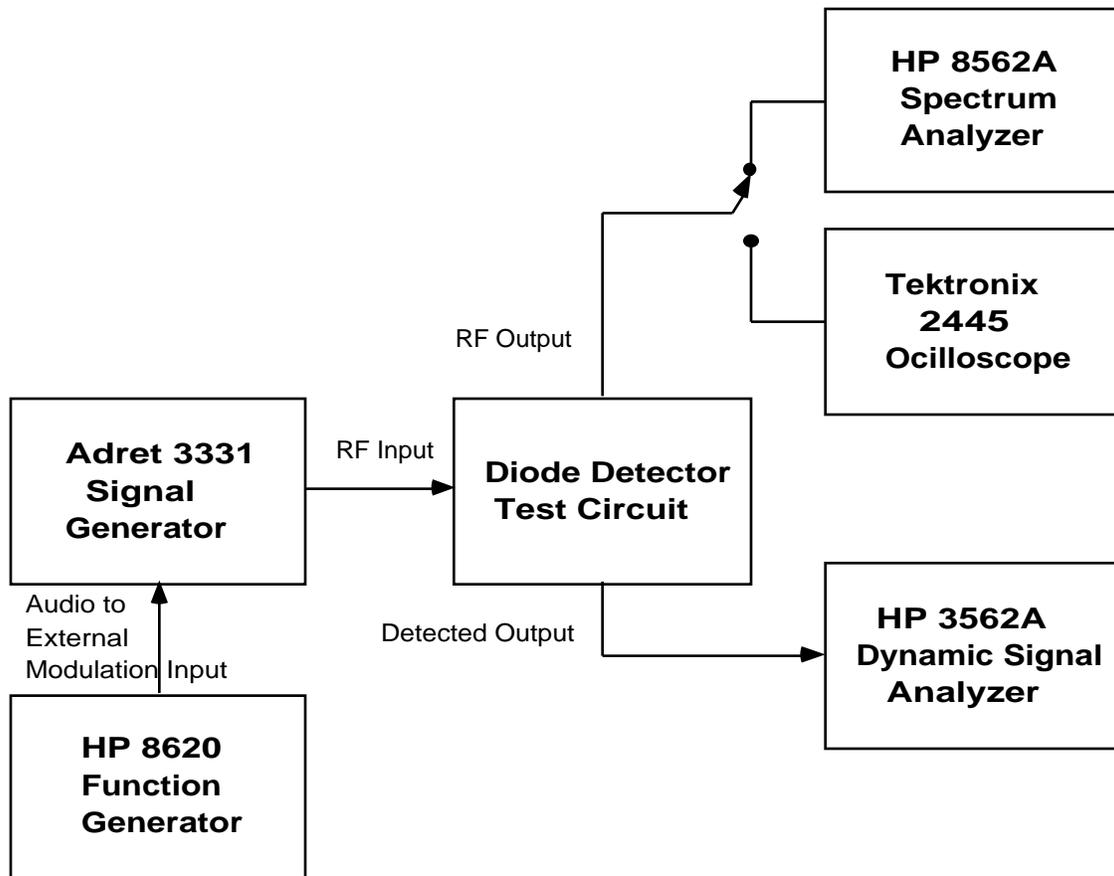


Figure 2 - Test Equipment Configuration

Modulation Index (%)	THD (%)
10	1.02
25	0.08
50	0.32
100	2.0
150	6.3

Table 2 - Measured Total Harmonic Distortion Versus Modulation Index

The THD measurements were made with the HP 3562A Dynamic Signal Analyzer. The 3562A is a two channel Fast Fourier Transform (FFT) based analyzer. It has a built in THD measurement function.

The measured THD values in Table 2 track closely with the calculated values in Table 1. The value for 10 percent modulation appears to be erroneous, as it is much greater than calculated. Also note that the THD values for 100 and 150 percent modulation are higher than the calculated values. This could be due to the contribution of the higher order harmonics in the measured values. These higher order components were not considered in the theoretical analysis. It is clear, though that distortion increases exponentially with modulation index or percentage. The distortion becomes problematic at indices of approaching 100 percent and severe at percentages greater than 100 percent.

Conclusion

Both theoretical and empirical analysis shows that the typical diode detector consisting of a series diode followed by a simple RC lowpass filter will produce a distorted output due to the inherent non-linearities of the diode. The distortion increases exponentially with the modulation index. The amount of distortion is most significant at modulation indices approaching and exceeding one.